Macroscopic Model of Geomagnetic-Radiation from Air Showers

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Abstract

We have developed a macroscopic description of coherent electro-magnetic radiation from air showers initiated by ultra-high energy cosmic rays in the presence of the geo-magnetic field. This description offers a simple and direct insight in the relation between the properties of the air shower and the time-structure of the radio pulse. As we show the structure of the pulse is a direct reflection of the important length scales in the shower.

Key words: Radio detection, Air showers, Cosmic rays, Geo-synchrotron, Geo-magnetic, Coherent radio emission
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1. Introduction

In recent years the interest in the use of radio detection for cosmic ray air showers is increasing with the promising results obtained from recent LOPES [1,2] and CODALEMA [3] experiments, which triggered plans to install an extensive array of radio detectors at the Pierre Auger Observatory [4]. For these reasons there is a strong interest to understand the link between the properties of the extensive air shower (EAS) and the time structure of the emitted pulse.

Already in the earliest works on radio emission from air showers [5,6,7,8], the importance of coherent emission was stressed. Two mechanisms, Cherenkov radiation and geo-magnetic radiation were investigated. In more recent work [9,10,11], the picture of coherent synchrotron radiation from secondary electrons and positrons in the Earth's magnetic field was proposed and extensive results on geo-synchrotron emission are given in [12].

The emission of an electromagnetic pulse by the charges in an EAS is described by Classical Electrodynamics [13]. In this sense it is an easy matter. The complication arises is the designation of all possible charge and current distributions that drive the electromagnetic emission process. For this models are needed such as the collective model described in Section 3. This macroscopic description of geomagnetic radiation [14,15] (MGMR) is in the basic idea very similar to that used in Ref. [7]. However, independent of any particular model, some of the general features of the emitted pulse can be calculated based the length scales involved in an EAS as is discussed in Section 2. For simplicity of the discussion we will limit ourselves here to an ideal case of a vertical airshower (along the z-axis) while the Earth-magnetic field is taken parallel to the Earth (along the y-axis). The case for more realistic geometry can be found in the literature [15].

2. A game of length scales

Our primary interest lies in the coherent emission of radio waves from an EAS. In a coherent process the amplitudes of the individual emitting charges interfere constructively such that the intensity of the radiation, the square of the amplitude, is proportional to the quare of the number of charges, N^2. Since the typical number of charges in an EAS is of the order of $N = 10^6$ it is clear that this process is far more important than incoherent emission where the intensity is just proportional to N.

Classical Electromagnetism [13] teaches that the coherent emission of a system of charges decreases linearly for wave-length which are considerably longer than the the size of the emitting body. For wave-length that are considerably shorter than that of the charge distribution the emission spectrum is truncated. The latter is due to destructive interference of radiation emitted from charges that are more than a wavelength apart. The length scales of the source thus put strong limitation on the frequency spectrum of the

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emitted radiation.

An EAS, independent how enormous the event may be, always exists for a limited time and occurs in a limited part of the atmosphere. The atmosphere it is assumed to be charge neutral free from electrical currents before the cosmic ray entered. The same applies to the atmosphere some time after the EAS has touched the ground. All electro dynamics thus occurs in a limited region of space-time and this has as an immediate consequence that the response of the system, the emitted radio waves, should vanish linearly in the limit of infinite wave length or zero frequency. Thus the time integral of the positive and negative parts of the emitted pulse should be equal. The simplest structure of the pulse is thus bi-polar but -in principle- it can have more than a single zero crossing.

The high frequency cut-off of the coherent response is at wave length that are shorter than the largest length scale of interest. Some of the length scales that may be important in this respect are i) the thickness of the front of the EAS, the pancake thickness, ii) the projected length of the EAS in the direction of the observer, iii) the lateral distribution of the charges in the EAS, and as last iv) something different from the previous three. Which length scale is determining the pulse shape is one of the challenges for models of radio emission from EAS. The high-frequency cut-off is reflected the short-time structure of the emitted pulse, the time between the start and the zero crossing.

3. The Macroscopic Model for Geo-Magnetic Radiation

The front of an EAS is formed by a plasma with copious amounts of electrons, positrons and other particles all moving towards the surface of the Earth with a velocity $c\beta_s$, almost the light velocity ($\beta_s \approx 1$). The magnetic field of the Earth induces, through the Lorentz force which pulls the electrons and positrons in opposite directions, a net electric current in the electron-positron plasma. This mechanism is very similar to that inducing an electric current in a copper wire that is moved through a magnetic field. The applied force induces a constant drift velocity due to collisions of the charge carriers with the air molecules (EAS) or copper atoms (wire), where the value depends on the strength of the magnetic field. Please notice that there are also large differences between electrons in metals, where the average velocity is due to thermal motion, and electrons in an EAS, where main component of the velocity is non-thermal an is attenuated with shower age. This macroscopic geomagnetic radiation model (MGMR) is discussed in detail in Ref. [14,15], here only the main findings are reproduced.

For the present estimate it is assumed that there are equal numbers of positive and negative charges moving towards the Earth with a large velocity. The number of electrons and positrons in the shower is parameterized as $N(z) = N_e f_t(t_r)$ in terms of the normalized shower profile, $f_t(t_r)$, at height $-z = c\beta_s t_r$ and where N_e is the number of electrons in the shower at the maximum. Due to the Earth's magnetic field a net electrical current in the $\hat{x}$-direction is induced with magnitude

$$j(x,y,z,t) = \langle v_d q\rangle \, e \, N_e f_t(t_r) \ , \tag{1}$$

where the pancake thickness is ignored. The drift velocity depends rather strongly on the model assumptions made [14,15] and we adopt a value of $\langle v_d q\rangle = 0.04$ c.

From the current density, j^μ, the vector potential is given by the Liénard-Wiechert fields,

$$A^\mu(x) = \frac{1}{4\pi\varepsilon_0} \int \left. \frac{j^\mu}{R(1-\vec{\beta}\cdot\hat{n})} \right|_{\mathrm{ret}} dh \ , \tag{2}$$

for a source with an infinitesimally small lateral extension. We use the common notation where $\hat{n}$ is a unit vector pointing from the source to the observer and R is the distance, both evaluated at the retarded time.

To obtain a simple estimate for the emitted radiation we take the limit $\beta_s = 1$ and $n = 1$ and ignore the thickness of the pancake. In this limit the, denominator in Eq. (2) can be rewritten to give

$$\begin{aligned} \mathcal{D} &= R(1-\vec{\beta_s}\cdot\hat{n})|_{\mathrm{ret}} = \sqrt{(-c\beta_s t + h)^2 + (1-\beta_s^2 n^2)d^2} \\ &= c\beta_s t + \mathcal{O}(1-\beta_s^2) \approx ct \ . \end{aligned} \tag{3}$$

Positive values of t correspond to negative retarded times,

$$ct_r = \frac{ct}{1+\beta_s} - \frac{d^2}{2c\beta_s t} + \mathcal{O}(1-\beta_s^2) \approx -\frac{d^2}{2ct} \ , \tag{4}$$

where $t = 0$ corresponds to the time the EAS touches Earth. From this equation we obtain the interesting observation that the earlier times of the received pulse is emitted at large (and negative) retarded times and thus large heights while the tail part of the pulse is emitted when the EAS was already close to the ground.

Since the current density has only an $\hat{x}$-component, the vector potential will share this property,

$$A^x(t,d) = J\frac{f_t(t_r)}{\mathcal{D}} \ , \tag{5}$$

where $J = \langle v_d q\rangle N_e e/4\pi\varepsilon_0 c$. The electric and magnetic fields are obtained from the vector potential by taking derivatives. Since in the present example the zeroth component of the vector potential vanished (no excess charge) the electric field is proportional to the time derivative of $\vec{A}$, giving

$$E_x(t,d) \approx J\frac{c^2 t_r^2 4}{d^4}\frac{d}{dt_r}[t_r f_t(t_r)] \ . \tag{6}$$

Since the vector potential vanishes at very early and (almost) vanishes at late retarded times, it is clear that the time-integrated electric field (being the derivative of A) also vanishes. This is in agreement with the observation made in Section 2.

One striking aspect of Eq. (6) is that the pulse is given as a simple function of t_r. Because of the relation Eq. (4)

the pulse at different distances is the same function of $t_r = -d^2/(2c^2t)$ i.e. at twice the distance from the point of impact it is four times as wide while the amplitude is decreased by a factor 2^4, see curves labeled 'appx' in Fig. 1.

Another interesting aspect is the realization that the zero crossing of the electric field corresponds to the maximum in the vector potential (in fact $t_r f_t(t_r)$). This part of the pulse thus corresponds to radio emission from the EAS at an height exceeding that of the shower maximum. The dominant part of the pulse is thus emitted at heights well above the shower maximum which implies that the radio signal has information on the early stages of the EAS development.

For small distances to the shower core, $d \approx ct$, the approximations made in deriving the expression for the retarded time, Eq. (4), are no longer valid and Eq. (6) is thus not applicable. More elaborate calculations [14] show that for observers close to the shower core, the pancake thickness become the important length scale, see blue curve in Fig. 1.

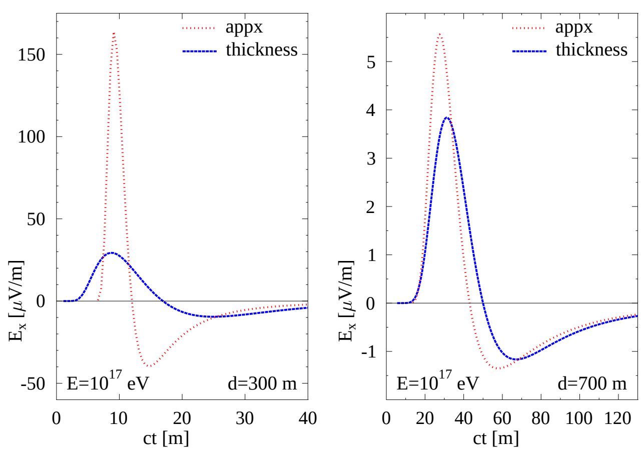

Fig. 1. [color online] *Predicted pulse shapes at* 300 *m and* 700 *m from the shower core for a* 10^{17} *eV shower. The dotted curve labeled 'appx' corresponds to the limiting case of Eq. (6) while the drawn curve includes the effects of the finite pancake thickness [14].*

4. Conclusions

We have shown that the macroscopic model for radio emission offers a very clear and simple picture for the emitted radio pulse. Different length scales determine the structure of the radio pulse, depending on the distance from the shower core. Close to the core the pulse length is determined by the pancake thickness while at larger distances the shower profile is reflected in the pulse structure. From the radio pulse at various distances from the shower core thus both the pancake thickness as well as the height of the shower maximum can be determined. This offers interesting prospects to determine the cosmic-ray composition.

The present discussion has been limited to the main contribution to the emitted signal coming from the electrical current induced by the Earth magnetic field. In Ref. [15] also the contribution are investigated which are due to induced electric dipole moments and charge excess for realistic EAS simulations. These are shown to give correction of the order of 10%, leaving the basic conclusions unchanged. Also the effect of more realistic geometries and a finite index of refraction of air have been studied.

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